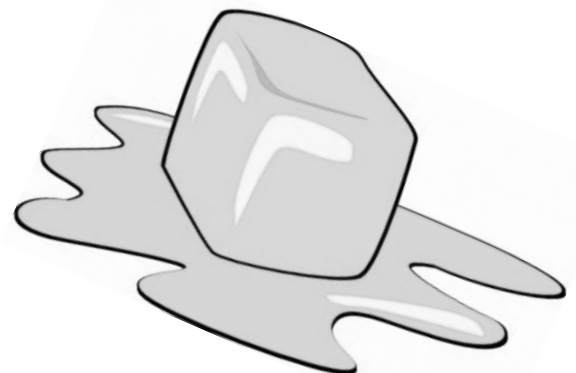


ALGEBRA 2 (COMMON CORE)



FACTS YOU MUST KNOW COLD FOR THE REGENTS EXAM



Notes to the Student

What is this? How do I use it to study?

Welcome to the “Algebra 2 (Common Core) Facts You Must Know Cold for the Regents Exam” study guide! I hope that you find this guide to be an invaluable resource as you are studying for your Algebra 2 Regents examination. This guide holds the essential information, formulas, and concepts that you *must know* in order to pass, or even master, your Regents exam! Over 150 hours have been put into the development of this study guide – from the clipart, to formatting, and to the mathematical theorems and concepts themselves, this packet has it all for you, the student and/or teacher! This study guide is specifically designed for students but can be used by teachers to ensure that there are no gaps in their curriculum. So, students, how do you use this to be incredibly successful? First and foremost, you need to *know this stuff cold*. There are no exceptions – you need to memorize and understand the material presented in this study guide. If you don’t know the basics, then how are you going to complete practice exams? You can’t. You need to take one step at a time; this is the first step. After you have read through these concepts and theorems several times, it’s time to try an administered Algebra 2 Regents exam. For your first attempt, I recommend that you have this study guide handy as a reference guide. If you’re stuck on a question, consult this guide to see what concept or theorem you need to apply to the problem. This method of getting stuck on a question, consulting this study guide, and finding the correct theorem helps your mind grow and retain these mathematical concepts. If you are still stuck, then visit www.nysmathregentsprep.com and watch our *fully explained* regents exam videos in Algebra 2. We have all exams available! I wish you all of mathematical success! If you have any questions, feel free to contact me at tclark@nysmathregentsprep.com. Good luck!



Notes to the Teacher

What’s new to this edition?

This is the second edition of the “Algebra 2 (Common Core) Facts You Must Know Cold for the Regents Exam”, published in the spring of 2018 as a **black & white friendly version**. If you are familiar with the previous version, you will immediately notice many changes. It was discovered that numerous topics were missing from the previous edition, hence this review packet has doubled in size. This may be a serious concern for some teachers due to paper and ink costs, but all topics in this new review packet *are important*. Each page was meticulously formatted. A listing shown below indicates the missing topics from the first edition that have been added in the second edition:

- ✓ Solving a system of equation with 3 variables
- ✓ Vertex form of a parabola with formulas of the focus and the directrix
- ✓ Transformation rules and descriptions using the acronym “Helicopters Do Rise Vertically” (HDRV)
- ✓ Rate of change/slope
- ✓ Exponential Growth and Decay formulas & Compound Interest formulas
- ✓ Regression equations using the TI-Nspire CX
- ✓ Equation of a circle and completing the square
- ✓ Revisions of the probability and statistics section

In addition to these topics, formatting was updated, diagrams were improved, and all typos that we were informed about were corrected. We hope that you find this study guide to be an invaluable resource for you and your students. We encourage you to make photo copies and distribute this to all of your students. If you teach other regents level courses such as Algebra 1 and Geometry, visit our website at www.nysmathregentsprep.com to download those study guides too! If you have any questions, comments, or suggestions, please don’t hesitate to contact me at tclark@nysmathregentsprep.com.

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Dedication

I would like to dedicate this study guide to the following mathematics teachers of Farmingdale High School, who have inspired me every step of the way to fulfil my goal of becoming a mathematics teacher: Mrs. Mary–Elena D’Ambrosio, Mrs. Laura Angelo–Provenza, Mrs. Louise Corcoran, Mrs. Efstratia Vouvoudakis, Mr. Scott Drucker, and Mr. Ed Papo. Other teachers who have also inspired me include Mrs. Jacquelyn Passante–Merlo and Mrs. Mary Ann DeRosa of W. E. Howitt Middle School, and Ms. Elizabeth Bove of Massapequa High School. I would especially like to thank Mrs. Mary–Elena D’Ambrosio for her suggestions and advice as to how to improve this second edition of the “Algebra 2 (Common Core) Facts You Must Know Cold for the Regents Exam”. Her input was invaluable.

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NUMBER SYSTEMS, POLYNOMIALS, & ALGEBRA

Dividing Polynomials

Division Algorithm: $\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$

Long Division of Polynomials

Steps:

- 1) Set up the problem, where $(x - a)$ is the divisor.
- 2) Divide the 1st term of the dividend by the 1st term of the divisor. Put the quotient above the 2nd term in the dividend.
- 3) Now, multiply the divisor by the quotient and write the product under the dividend, properly aligning the terms. Now subtract this product from the dividend, and a term should cancel.
- 4) Repeat the same process, taking into account the locations of the monomials.
- 5) Write your final answer.

Example: $(2x^2 + 7x + 6) \div (x + 2)$

$$\begin{array}{r}
 2x + 3 \\
 x + 2 \overline{) 2x^2 + 7x + 6} \\
 \underline{2x^2 + 4x} \quad \text{(subtract)} \\
 3x + 6 \\
 \underline{3x + 6} \quad \text{(subtract)} \\
 0 \quad \text{remainder}
 \end{array}$$

2x was needed to create the first term of $2x^2$

Answer: $2x + 3$

Synthetic Division of Polynomials

Steps:

- 1) First, analyze if the divisor is in the form of $(x - a)$, where x has a leading coefficient of 1. If not, and it's in the form of $(\beta x - a) | \beta \in \mathbb{Z}$, you *must* divide by β at the conclusion of the problem!
- 2) Set $x - a = 0$, and put a in the “window”
- 3) Line up the coefficients of the dividend.
Warning – do *not* skip powers!
- 4) Bring down the first coefficient, and multiply this number by the value of a . Write this number under the second number in the dividend. Repeat the process
- 5) Write your answer in standard form using the resulting coefficients, reducing each power by one degree.

Example: $(x^3 + 6x^2 + 7x - 6) \div (x + 4)$

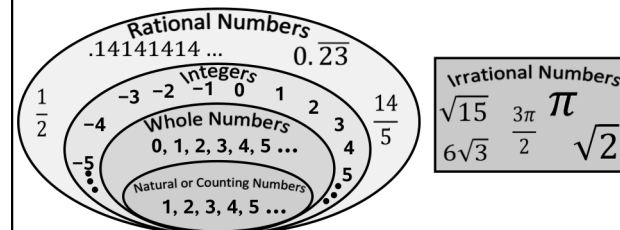
$$\begin{array}{r}
 x + 4 = 0 \\
 -4 \quad -4 \\
 \hline
 \boxed{x = -4} \quad \begin{array}{c} \text{multiply} \\ \downarrow \\ \boxed{-4} \end{array}
 \end{array}$$

$$\begin{array}{r}
 1 \quad 6 \quad 7 \quad -6 \\
 \downarrow \quad \text{Add} \quad \text{Add} \quad \text{Add} \\
 1 \quad 2 \quad -1 \quad -2 \\
 \hline
 \text{Remainder}
 \end{array}$$

Answer: $x^2 + 2x - 1 + \frac{-2}{x+4}$

Quick Review of the Real Number System (Denoted as \mathbb{R})

The Real Number System



The Remainder Theorem

When the polynomial $f(x)$ is divided by a binomial in the form of $(x - a)$, the remainder equals $f(a)$.

$$\frac{4x^2 + 2x - 5}{(x - 1)}$$

$$f(1) = 4(1)^2 + 2(1) - 5 \Rightarrow 1$$

The remainder is 1!

The Factor Theorem

If $f(a) = 0$ for polynomial $f(x)$, then a binomial in the form of $(x - a)$ must be a factor of the polynomial.

$$\frac{x^4 + 6x^3 + 7x^2 - 6x - 8}{(x + 4)}$$

$$\begin{aligned}
 f(-4) &= (-4)^4 + 6(-4)^3 + 7(-4)^2 - 6(-4) - 8 \\
 f(-4) &= 256 + (-384) + 112 - (-24) - 8 \\
 f(-4) &= 0
 \end{aligned}$$

The remainder is zero, therefore $(x + 4)$ is a factor!



Factoring

The Order of Factoring:

Greatest Common Factor (GCF) → Difference of Two Perfect Squares (DOTS) → Trinomial (TRI) → "AC" Method / Earmuff Method (AC)

GCF: $ab + ac = a(b + c)$

DOTS: $x^2 - y^2 = (x + y)(x - y)$

TRI: $x^2 - x + 6$
 $\Rightarrow (x + 2)(x - 3)$

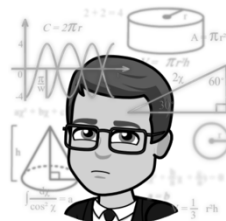
AC ($a \neq 1$): $2x^2 + 15x + 18$
 $x^2 + 15x + 36$

... and if all else fails to find the roots of a *quadratic* (an equation with an x^2 term), use the

→ Quadratic Formula (QF):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Keep in mind that this formula is on your reference sheet, but you should really memorize it!



$(x + 12)(x + 3)$

$(x + \frac{12}{2})(x + \frac{3}{2})$

$(x + 6)(2x + 3)$

Other Forms of Complex Factoring

Factor by Grouping:

$$\underbrace{x^3 + 2x^2}_{\text{Common factor of } x^2} - \underbrace{3x - 6}_{\text{Common factor of } -3}$$

$\Rightarrow x^2(x + 2) - 3(x + 2)$
 $\Rightarrow (x^2 - 3)(x + 2)$

Steps

- 1) Group the first two terms and the last two terms. Re-arrange the original polynomial if necessary
- 2) Factor out GCF in both; the resulting binomial must be the same
- 3) Simply and write in correct form

Factoring Perfect Cubes

by SOAP:

$x^3 - 8$

$(x)^3 - (2)^3$

$(x - 2)(x^2 + 2x + 4)$

Steps

- S – "Same" as the sign in the middle of the original expression
 O – "Opposite" sign
 AP – "Always Positive"
 1) Take the cube root of each term
 2) Write this result as a binomial, then find the trinomial using the first and last terms as a reference.

Rational Expressions & Equations

➤ To add or subtract rational expressions, you need to find a *common denominator!*

$$\frac{10}{2x^2} + \frac{5}{3x} \Rightarrow \frac{3}{3} \cdot \frac{10}{2x^2} + \frac{5}{3x} \cdot \frac{2x}{2x} \Rightarrow \frac{30}{6x^2} + \frac{10x}{6x^2} = \frac{30 + 10x}{6x^2}$$

➤ To multiply rational expressions, factor first, reduce, and then multiply through.

$$\frac{6a}{3a + 15} \cdot \frac{4a + 20}{2a^2} \Rightarrow \frac{\overset{2}{\cancel{6a}}}{\underset{1}{3(a+5)}} \cdot \frac{\overset{2}{\cancel{4(a+5)}}}{\underset{1}{2a^2 \cdot \cancel{a}}} \Rightarrow \frac{2}{1} \cdot \frac{2}{a} = \frac{4}{a}$$

➤ To divide rational expressions, flip the second fraction, factor, reduce, and then multiply through.

$$\frac{6x + 18}{4} \div \frac{x^2 + 3x}{5x^2} \Rightarrow \frac{6x + 18}{4} \cdot \frac{5x^2}{x^2 + 3x} \Rightarrow \frac{\overset{3}{\cancel{6(x+3)}}}{\underset{2}{\cancel{4}}} \cdot \frac{\overset{5}{\cancel{5x^2}} \cdot \cancel{x}}{\cancel{x(x+3)}} = \frac{15x}{2}$$

➤ Complex Fractions: Multiply each fraction by the LCD, cancel what's common. & simplify.

$$\frac{\overset{x^2}{\cancel{x^2}} \cdot \frac{2}{\cancel{x^2}} - \frac{4}{\cancel{x}} \cdot \frac{x}{\cancel{x^2}}}{\overset{x^2}{\cancel{x^2}} \cdot \frac{4}{\cancel{x}} - \frac{2}{\cancel{x^2}} \cdot \frac{2}{\cancel{x^2}}} \Rightarrow \frac{2 - 4x}{4x - 2} = -1$$

➤ Solving Rational Equations: Find a common denominator, multiply each fraction *only by what is "needed"*, solve for the equation in the numerator. **Check answers when complete!**

$$\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x} \Rightarrow \frac{3}{3} \cdot \frac{1}{x} - \frac{1}{3} \cdot \frac{1}{1} \cdot \frac{x}{x} = -\frac{1}{3x} \Rightarrow \frac{3}{3x} - \frac{x}{3x} = -\frac{1}{3x}$$

$3 - x = -1 \Rightarrow x = 4$



Properties of Exponents & Radicals

$$x^0 = 1$$

$$x^m \cdot x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(xy)^n = x^n \cdot y^n$$

$$x^{-m} = \frac{1}{x^m}$$

$$(x^n)^m = x^{n \cdot m}$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{\frac{p}{r}} = \sqrt[r]{x^p}$$

$$\sqrt[a]{x} = x^{\frac{1}{a}}$$

$$\sqrt[n]{a^n} = a$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Rationalization of Denominators in the Form of $a \pm \sqrt{b}$

Steps:

- 1) Begin by analyzing if the denominator contains a radical
- 2) If the denominator has a radical, multiply both the top and bottom of the fraction by the *conjugate* of the denominator (simply switch the sign in the middle of a and b). Then simplify as much as possible

Example: Simplify $\frac{5}{2+\sqrt{3}}$

$$\frac{5}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} \Rightarrow \frac{5(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} \Rightarrow \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}+\sqrt{9}} \Rightarrow 10-5\sqrt{3}$$

Exponentials & Logarithms

Exponential Form $\rightarrow B^e = N$ \Leftrightarrow $\log_B N = e$ \leftarrow Logarithmic Form
 ... where B is the "base", e is the "exponent, and N is the "number".

An exponent and a logarithm are inverses of each other!

Properties of Logarithms

$$\log_b(m \cdot n) = \log_b m + \log_b n$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

$$\log_b m^r = r \log_b m$$

$$\log_b b = 1$$

$$\log_b 1 = 0$$

Properties of Natural Logarithms

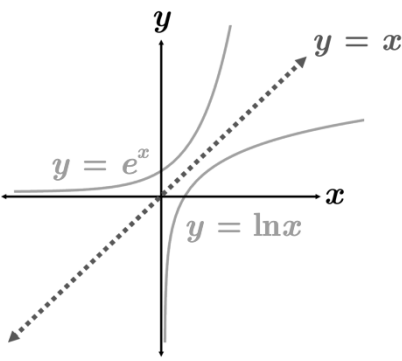
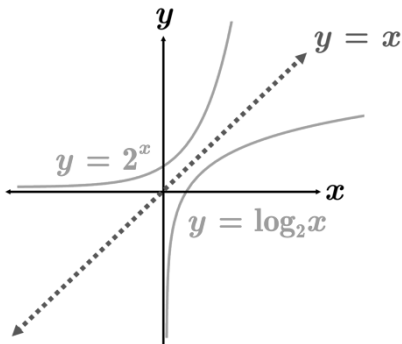
$$\ln(ab) = \ln a + \ln b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\ln a^b = b \ln a$$

$$\ln 1 = 0$$

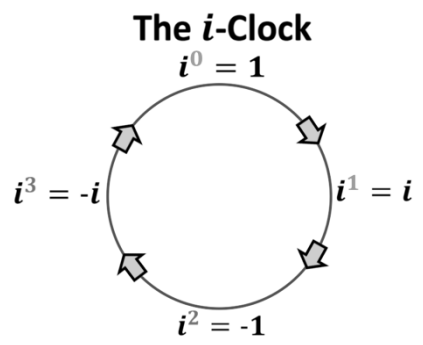
$$\ln e = 1$$



Complex Numbers

The imaginary unit, i , is the number whose square is negative one.

$$\sqrt{-1} = i \Leftrightarrow i^2 = -1$$



To solve for a value of i , you can use your calculator or you can use the i -clock!

Example: Solve for i^7

To solve, start at the top (i^0) and count around the clock at each quarter interval, and stop when you reach i^7 . The answer is $-i$.

Rationalization of Denominators in the Form of $a \pm bi$

Steps:

- 1) Begin by analyzing if the denominator contains a power of i
- 2) If the denominator has i , multiply both the top and bottom of the fraction by the *conjugate* of the denominator (simply switch the sign in the middle of a and b). Then simplify as much as possible

Example: Simplify $\frac{4+i}{2-5i}$

$$\frac{4+i}{2-5i} \cdot \frac{(2+5i)}{(2+5i)} \Rightarrow \frac{8+20i+2i+5i^2}{4+10i-10i-25i^2} \Rightarrow \frac{8+22i+5(-1)}{4-25(-1)} \Rightarrow \frac{3+22i}{29}$$



FUNCTIONS

Definition: A **function** is a relation that consists of a set of ordered pairs in which each value of x is connected to a unique value of y based on the rule of the function. For each x value, there is one and only one corresponding value of y .

Domain & Range

Domain: The largest set of elements available for the independent variable; the first member of the ordered pair (x).

Note: Unless otherwise noted, the domain is the set of real numbers, denoted as \mathbb{R} . There are however scenarios where the domain is *restricted*. This occurs when radicals and fractions arise.

Restrictions on Domain

1. **Fraction:** The denominator cannot be zero.

Set the entire denominator equal to zero and solve.

$$f(x) = \frac{x-4}{x+3}; x+3=0 \Rightarrow x \neq -3 \Rightarrow \mathbf{D: \{ \mathbb{R} | x \neq -3 \}}$$

2. **Radical:** The radicand cannot be negative.

Set the radicand greater than or equal to zero and solve.

$$f(x) = \sqrt{x-5}; x-5 \geq 0 \Rightarrow x \geq 5 \Rightarrow \mathbf{D: \{ \mathbb{R} | x \geq 5 \}}$$

3. **Radical in the Denominator:** The radical cannot be negative *and* the denominator cannot be zero.

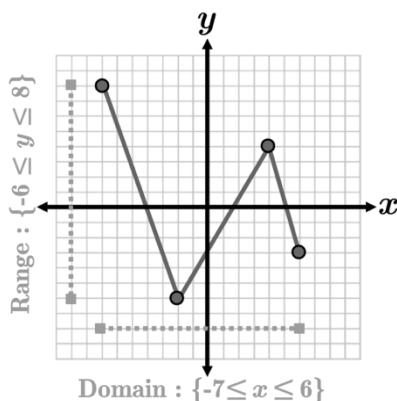
Set the radicand greater than zero and solve.

$$f(x) = \frac{1}{\sqrt{x+7}}; x+7 > 0 \Rightarrow x > -7 \Rightarrow \mathbf{D: \{ \mathbb{R} | x > -7 \}}$$

Range: The set of elements for the dependent variable, the second member of the ordered pair (y).

Note: Unless otherwise noted, the range is the set of real numbers, denoted as \mathbb{R} . In this course, you are *not* expected to find the range algebraically. ☺

Graphical Example:

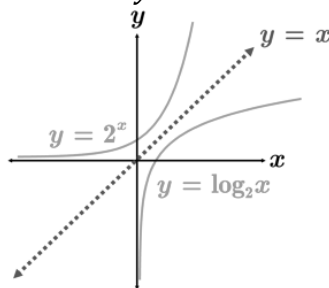


Inverse Functions

The inverse of a function is the reflection of the function over the line $y = x$. Only a one-to-one function has an inverse function. To solve for an inverse given a function $f(x)$, switch the places of x and y , then solve for y .

Notation:

$f(x)$ is the **function**
 $f^{-1}(x)$ is the **inverse**



Composition Functions: One function is substituted into another in place of the variable. This can involve numeric substitutions or substitutions of an algebraic expression in the function in the place of the variable.

Notation: $f(g(x))$ or $f \circ g(x)$

★ **Always read from right to left when using this notation.** ★

Example 1: If $f(x) = x + 9$ and $g(x) = 2x + 3$, find $f(g(3))$.

$$g(3) = 2(3) + 3 \Rightarrow 6 + 3 = 9$$

$$f(9) = (9) + 9 = 18 \Rightarrow \text{Answer: } f(g(3)) = \mathbf{18}$$

Example 2: If $f(x) = x + 5$ and $g(x) = 3x + 4$, find $(f \circ g)(x)$

$$g(x) = 3x + 4$$

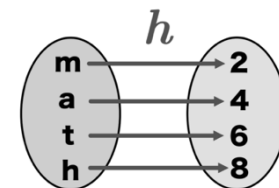
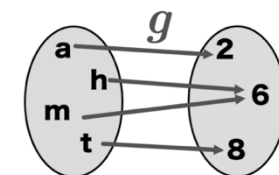
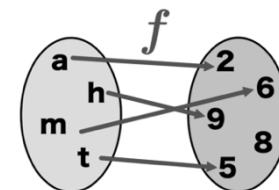
$$f(3x + 4) = (3x + 4) + 5 \Rightarrow 3x + 9 \Rightarrow \text{Answer: } (f \circ g)(x) = \mathbf{3x + 9}$$

One-to-One & Onto Functions

One-to-One Function: A one-to-one function must be a function, where when the ordered pairs are examined, there are no repeating x values or y values. One-to-one functions also pass *both* the horizontal and vertical line tests.

Onto Function: A mapping, $g : A \rightarrow B$ in which each element of set B is the image of at least one element in set A . In other words, all x values and all y values are used.

One-to-One & Onto Function: A function where all x values and all y values are used, where none of the x or y values repeat themselves. This function must also pass the horizontal and vertical line tests.



End Behavior

The end behavior of a graph is defined as the **direction the function is heading at the ends of the graph**. The end behavior can be determined by the following:

1. The degree of the function
2. The leading coefficient of the function

Notation:

$$\text{As } x \rightarrow \pm\infty, f(x) \rightarrow \pm\infty$$

This notation is read as, "As x approaches positive/negative infinity, y approaches positive/negative infinity."

(*NOTE*: In Algebra 2, these are the only two notations you should know for end behavior.)

Multiplicity

Multiplicity is defined as how many times a unique root repeats itself based on the characteristics of the graph, as it pertains to the x -axis, or based on the polynomial equation itself.

Polynomial Characteristics:

Multiplicity of 1

The equation $y = x + 1$ has the root $x = -1$, and is unique only once.

Multiplicity of 2

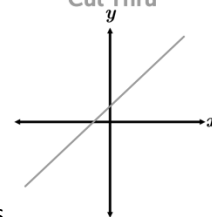
The equation $y = x^2$ has the root $x = 0$, and is unique and repeats itself two times.

Multiplicity of 3

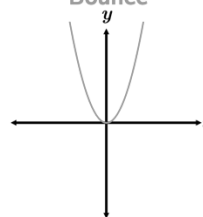
The equation $y = x^3$ has the root $x = 0$, and is unique and repeats itself three times.

Graphical Characteristics:

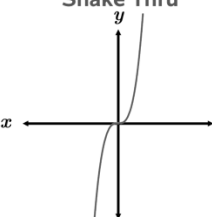
Multiplicity of 1
"Cut Thru"



Multiplicity of 2
"Bounce"



Multiplicity of 3
"Snake Thru"

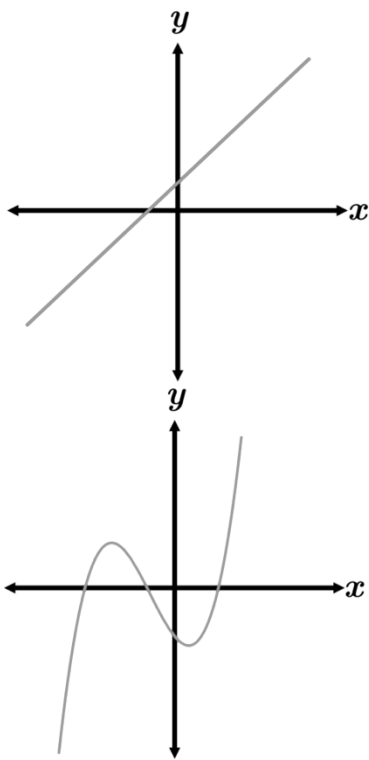


Odd Degree Polynomials

Positive Leading Coefficient

$$\text{As } x \rightarrow -\infty, f(x) \rightarrow -\infty$$

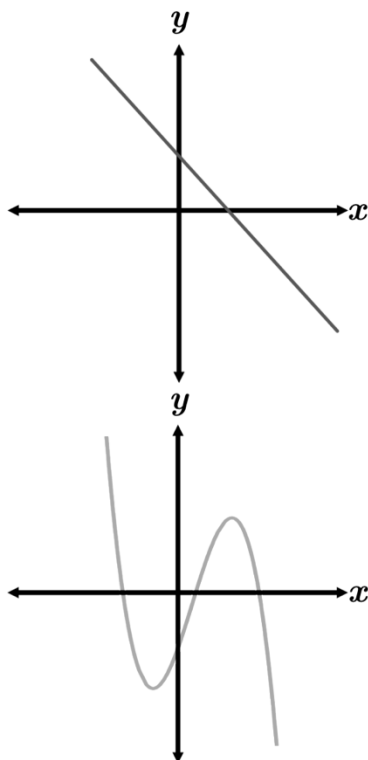
$$\text{As } x \rightarrow \infty, f(x) \rightarrow \infty$$



Negative Leading Coefficient

$$\text{As } x \rightarrow -\infty, f(x) \rightarrow \infty$$

$$\text{As } x \rightarrow \infty, f(x) \rightarrow -\infty$$

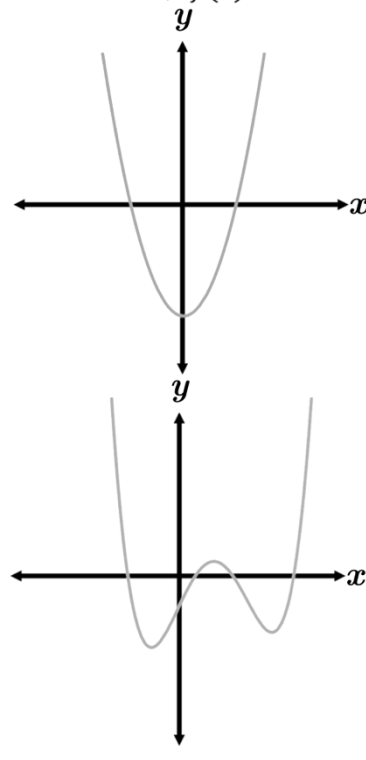


Even Degree Polynomials

Positive Leading Coefficient

$$\text{As } x \rightarrow -\infty, f(x) \rightarrow \infty$$

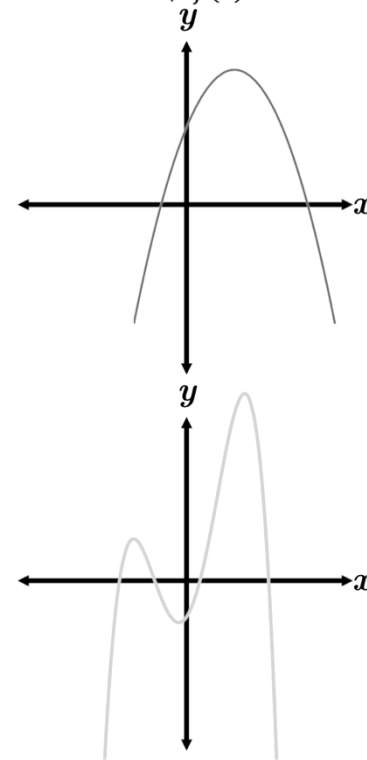
$$\text{As } x \rightarrow \infty, f(x) \rightarrow \infty$$



Negative Leading Coefficient

$$\text{As } x \rightarrow -\infty, f(x) \rightarrow -\infty$$

$$\text{As } x \rightarrow \infty, f(x) \rightarrow -\infty$$



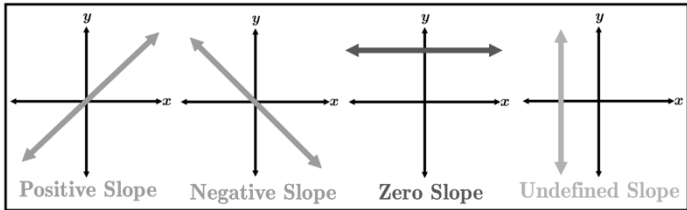
LINEAR SYSTEMS & EQUATIONS

Definition: A *linear equation* has a variable, usually written as x , with a degree of one.

Review of Slope & Equations of Lines

Slope Formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Diagrams of Slopes:



Slope-Intercept Form of a Line: $y = mx + b$

where m is the slope and b is the y -intercept

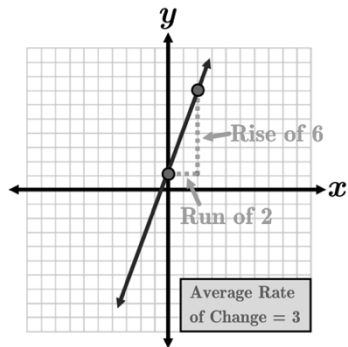
Point-Slope Form of a Line: $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a given point on the line

On the Algebra 2 Regents, you will no longer see the word "slope"; rather, you will be asked to calculate the **average rate of change**, which we will now define.

Definition: For a function $y = f(x)$ between the values $x = a$ and $x = b$, the average rate of change is defined as

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Look familiar? That's right! The word **slope** means the **exact same thing** as the phrase **average rate of change**.



Solving a System of Linear Equations with 3 Variables

At this level, you are expected to know how to solve a system of linear equations with two variables, usually x and y . If you need to review this, take a look at the same study guide for Algebra 1. We will now analyze how to solve a system of linear equations with three variables by listing general steps, followed by an example.

Steps:

- 1) Begin by grouping the 1st and 2nd equations, and the 2nd and 3rd equations, respectively.
- 2) Now that we have two different systems of linear equations with three variables, eliminate **one** of the **same** variables from both systems.
- 3) Once a variable is eliminated, simplify to one equation in both systems.
- 4) Now you should have one equation from each; combine these two equations to create a new system of equations.
- 5) Eliminate another variable and solve for the remaining variable.
- 6) Once you find one variable, back-substitute to find the other two, choosing your equations to substitute into wisely. 😊

Example: Solve

$$\begin{array}{l} x + y + z = 1 \\ 2x + 4y + 6z = 2 \\ -x + 3y - 5z = 11 \end{array}$$

$$\begin{array}{r} -2(x + y + z = 1) \\ 2x + 4y + 6z = 2 \\ \hline -2y - 2z = -2 \\ + 4y + 6z = 2 \\ \hline 2y + 4z = 0 \end{array}$$

$$\begin{array}{r} 2x + 4y + 6z = 2 \\ -x + 3y - 5z = 11 \\ \hline 4y + 6z = 2 \\ + 6y - 10z = 22 \\ \hline 10y - 4z = 24 \end{array}$$

$$\begin{array}{r} 2y + 4z = 0 \\ 10y - 4z = 24 \\ \hline 12y = 24 \\ \frac{12}{12} \frac{y}{1} = \frac{24}{12} \\ y = 2 \end{array}$$

Solve z by Substitution

$$\begin{array}{l} 4y + 6z = 2 \\ 4(2) + 6z = 2 \\ 8 + 6z = 2 \\ z = -1 \end{array}$$

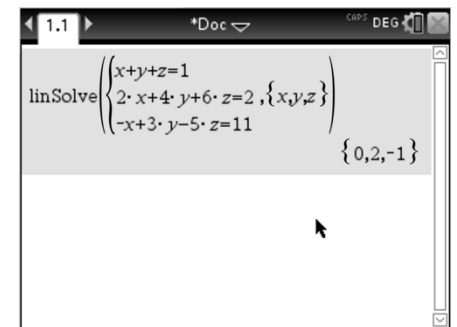
Solve x by Substitution

$$\begin{array}{l} x + y + z = 1 \\ x + (2) + (-1) = 1 \\ x = 0 \end{array}$$

Answer: $\{0, 2, -1\}$



Still don't get it? It can be confusing... but if you have the TI-Nspire CX calculator, you can get the answer! Check out our TI-Nspire CX Guide for Algebra 2 (Common Core) for the procedure.



QUADRATIC SYSTEMS & CONIC SECTIONS

Definition: A *quadratic equation* is a polynomial equation with a degree of two, usually containing an “ x^2 ” term.

The Standard Form of a Quadratic

The standard form of a quadratic is in the form of

$$ax^2 + bx + c = 0$$

where a, b , and c are constants where $a \neq 0$.

The Sum of the Roots of a Quadratic

Sum of the Roots: $r_1 + r_2 = \frac{-b}{a}$

where a and b are constants from a quadratic equation in the form of $ax^2 + bx + c = 0$.

The Product of the Roots of a Quadratic

Product of the Roots: $r_1 \cdot r_2 = \frac{c}{a}$

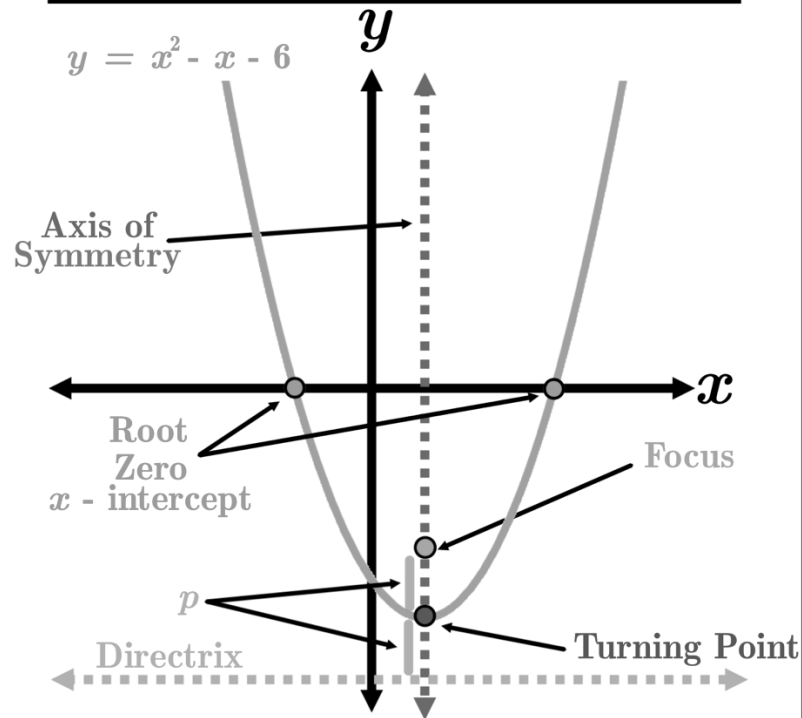
where a and c are constants from a quadratic equation in the form of $ax^2 + bx + c = 0$.

Writing the Equation of a Quadratic Given Two Roots

If you are given two roots as r_1 and r_2 , then a quadratic equation can be written in the form of

$$x^2 - \text{sum}x + \text{product} = 0 \Leftrightarrow x^2 - (r_1 + r_2)x + (r_1 \cdot r_2) = 0$$

The Parts of a Quadratic



Definitions

Root/Zero/x-intercept: a point on a quadratic where $f(x) = 0$. It is a point where the quadratic intersects the x -axis, so long as the root $x \in \mathbb{R}$. If $x \in \mathbb{C} \mid \mathbb{R} \not\subseteq \mathbb{C}$, the root does not intersect the x -axis.

Turning Point: also called a **vertex**, it's the point on a quadratic where the direction of the function changes.

Axis of Symmetry: a line of symmetry in the form of $x = c$, where c is a constant. The value of c is the *same value* as the x value of the turning point.

Focus: a point which lies inside the parabola and on the axis of symmetry. It is some distance away from the turning point of the parabola, denoted as p .

Directrix: a line that is perpendicular to the axis of symmetry & lies outside the parabola. It is some distance away from the turning point of the parabola, denoted as p .

Important Formulas for Quadratics

Axis of Symmetry: $x = \frac{-b}{2a}$

Vertex of a Parabola: (h, k)

Vertex Form of a Parabola:

$$y = \frac{1}{4p}(x \pm h)^2 \pm k$$

Focus: $(h, k + p)$

Directrix: $y = k - p$

Finding the Equations of the Focus & Directrix – Example

The directrix of a parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Let's first re-write the given quadratic in vertex form:

$$12(y + 3) = (x - 4)^2 \Rightarrow y + 3 = \frac{1}{12}(x - 4)^2 \Rightarrow y = \frac{1}{4(3)}(x - 4)^2 - 3$$

Based on this equation, we can see that the vertex is $(4, -3) \Rightarrow h = 4, k = -3$. We know the equation of the directrix is $y = -6$. We can re-write this as $-6 = k - p$. We know that $k = -3$ from above, so from this equation, $p = 3$. The focus is $(h, k + p) \Rightarrow (4, (-3) + (3)) \Rightarrow (4, 0)$

(It is true that the value of p can be found from the vertex form of the parabola, but we decided to show the extra step.)



The Discriminant

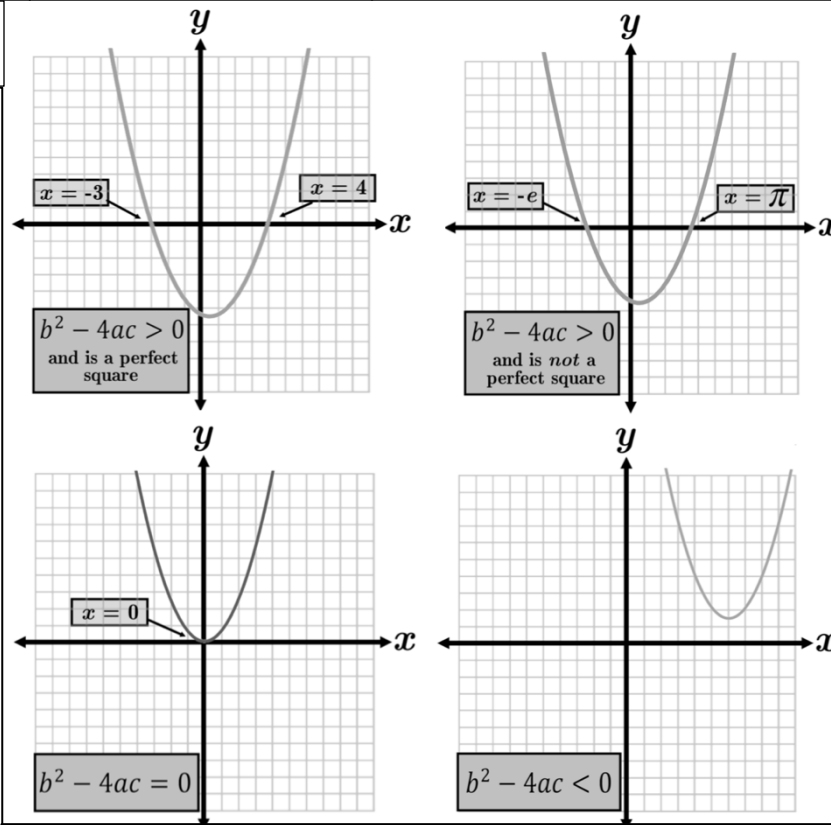
The discriminant is a part of the quadratic formula which allows mathematicians (and students!) to anticipate the nature of the roots. In other words, it determines what kinds of roots a particular quadratic equation will have. The formula is:

$$b^2 - 4ac$$

where a , b , and c are constants

The Value of the Discriminant	The Nature of the Roots	Number of x -Intercepts
$b^2 - 4ac > 0$, and is a perfect square	Real, Rational, & Unequal	2
$b^2 - 4ac > 0$, and is <i>not</i> a perfect square	Real, Irrational, & Unequal	2
$b^2 - 4ac = 0$	Real, Rational, & Equal	1 (multiplicity of 2, called a <i>bounce</i>)
$b^2 - 4ac < 0$	Imaginary	0 (never touches the x -axis)

Diagrams of Different Discriminant Values



Completing the Square

The method of “completing the square” is used when factoring by the basic “Trinomial Method”, or “AM” method cannot be applied to the problem. The completing the square method is commonly used in geometry to express a **general circle equation in center-radius form**.

Example: Express the general equation $4x^2 - 24x + 4y^2 + 72y - 76 = 0$ in center-radius form.

$$4(x^2 - 6x + y^2 + 18y - 19 = 0)$$

$$4(x^2 - 6x + y^2 + 18y = 19)$$

$$4(x^2 - 6x + _ + y^2 + 18y + _ = 19 + _ + _)$$

$$4(x^2 - 6x + 9 + y^2 + 18y + 81 = 19 + 9 + 81)$$

$$4((x - 3)^2 + (y + 9)^2 = 109)$$

$$4(x - 3)^2 + 4(y + 9)^2 = 436$$

Formula: $\left(\frac{b}{2}\right)^2$

Steps:

- 1) Determine if the squared terms have a coefficient of 1. If not, you need to factor out whatever number is there.
- 2) If there is a constant/number on the left side of the equal sign, move that constant to the right side
- 3) Insert “boxes” or “blank spaces” after the linear terms to acquire a perfect-square trinomial
- 4) Take half of the linear term(s) and square the number. Insert this number on both the left and right sides
- 5) Factor using the “trinomial method”
- 6) Write your equation and redistribute the factor if necessary.

Graphing Circles

Steps:

- 1) Determine the center and the radius
- 2) Plot the center on the graph
- 3) Around the center, create four loci points that are equidistant from the center of the circle
- 4) Using a compass or steady freehand, connect all four points. Label when finished



TRIGONOMETRY & TRIGONOMETRIC FUNCTIONS

Reciprocal & Quotient Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Radians

To change from *degrees* to *radians*, multiply by

$$\frac{\pi}{180}$$

Degrees

To change from *radians* to *degrees*, multiply by

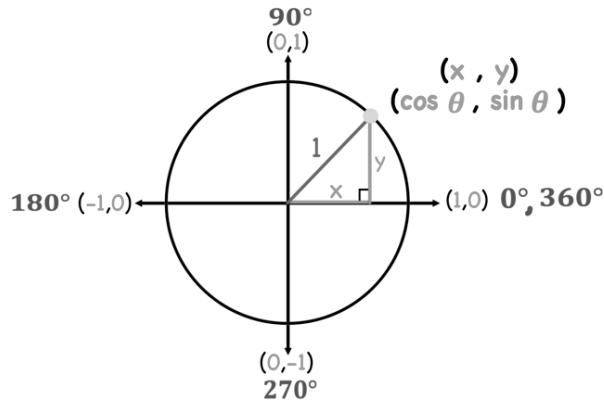
$$\frac{180}{\pi}$$

Trigonometric Functions

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} \quad \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} \quad \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

The Unit Circle



The Unit Circle – Exact Values

Remember the table below!

$$\cos \theta = x \quad \tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

$$\sin \theta = y$$

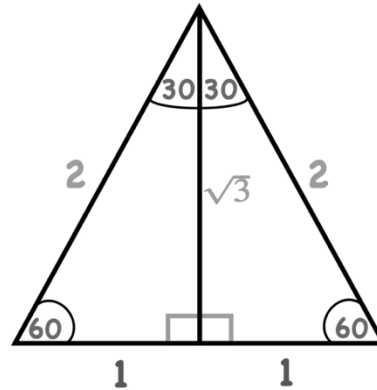
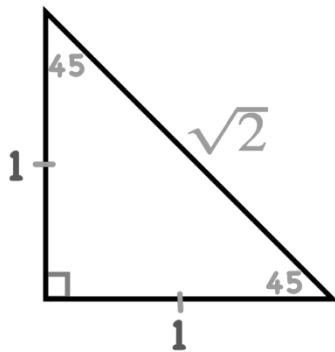
θ	0°	90°	180°	270°	360°
$\sin \theta$	0	1	0	-1	0
$\cos \theta$	1	0	-1	0	1
$\tan \theta$	0	UNDEF	0	UNDEF	0

Arc Length of a Sector

$$s = r \cdot \theta$$

where s is the length of the sector, r is the length of the radius, and θ is an angle in radians.

Special Right Triangles



Special Right Triangles – Exact Values

Remember the table below!

θ	30°	45°	60°
$\sin \theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan \theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

The Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

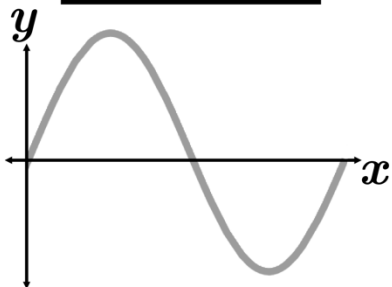
Inverse Notations

- The inverse of $y = \sin x$ is $y = \sin^{-1} x$ or $y = \arcsin(x)$
- The inverse of $y = \cos x$ is $y = \cos^{-1} x$ or $y = \arccos(x)$
- The inverse of $y = \tan x$ is $y = \tan^{-1} x$ or $y = \arctan(x)$



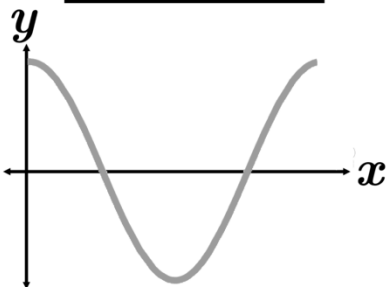
Trigonometric Graphs & Equations

THE SINE CURVE



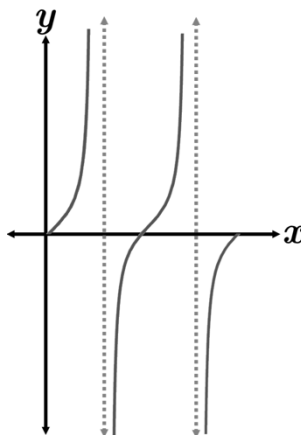
$$y = A \sin(B(x - C)) + D$$

THE COSINE CURVE



$$y = A \cos(B(x - C)) + D$$

THE TANGENT CURVE



$$y = A \tan(B(x - C)) + D$$

Important Formulas & Definitions

Amplitude (A): The vertical distance between the midline and one of the extremum points.

$$\text{Formula: } \frac{1}{2} | \text{Maximum} - \text{Minimum} |$$

Frequency (B): The number cycles the graph completes in 2π radians.

Horizontal Shift (C): The movement of a function left or right. The sign used in the equation is opposite the direction in which the function moves.

Vertical Shift (D): The movement of a function up or down. The sign used in the equation is the same direction in which the function moves.

Period: The horizontal length to complete one complete cycle.

$$\text{Formula: } \frac{2\pi}{b}, \text{ where } b \text{ is the frequency}$$

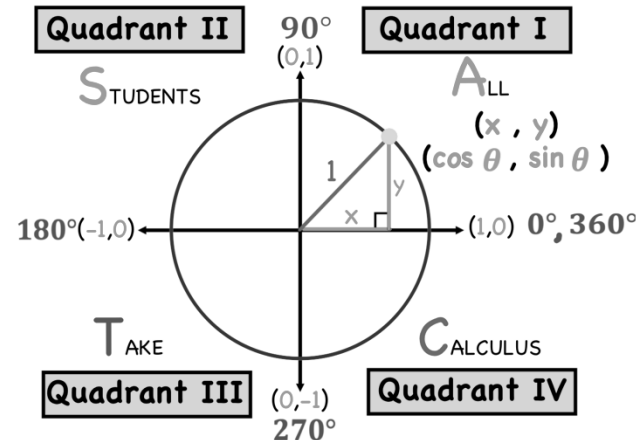
Sketch Point (Optional): Tells you where and how often to plot points.

$$\text{Formula: } \frac{\text{Period}}{4}$$

Midline/Vertical Shift: The horizontal line that passes exactly in the middle between the graph's maximum and minimum points.

$$\text{Formula: } \frac{1}{2} | \text{Maximum} + \text{Minimum} |$$

The Quadrants & Trigonometric Relationships



QUADRANT I: All trigonometric functions are positive

QUADRANT II: Only sine and cosecant are positive

QUADRANT III: Only tangent and cotangent are positive

QUADRANT IV: Only cosine and secant are positive

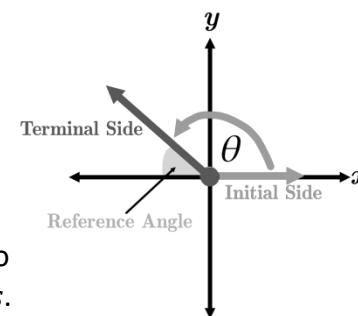
Reference Angles & Drawing in Standard Position

Standard Position: an angle whose initial side is on the positive x - axis.

Initial Side: the ray of an angle that is the starting place for the rotation of the angle.

Terminal Side: the ray that is rotated to the location that shows the measure of the angle.

Reference Angle: measured from the terminal side of the main angle to the closest x - axis.



TRANSFORMATIONS OF FUNCTIONS

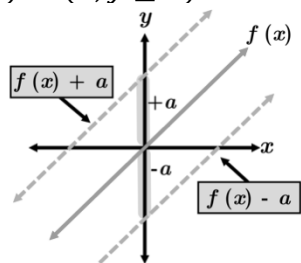
Definition: A *transformation* refers to the movement (translation, reflection, rotation), or dilation of an object or a function around the coordinate plane.

Summary of Transformation Rules

Vertical Translation:

Function: $f(x) \rightarrow f(x) \pm a$

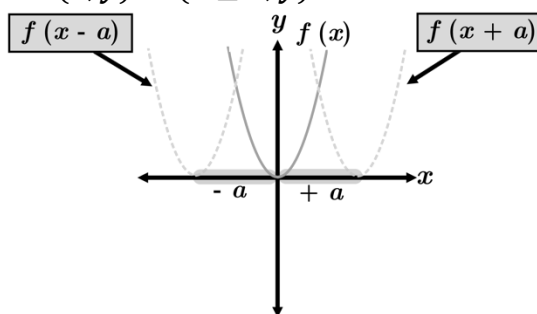
Point: $(x, y) \rightarrow (x, y \pm a)$



Horizontal Translation:

Function: $f(x) \rightarrow f(x \pm a)$

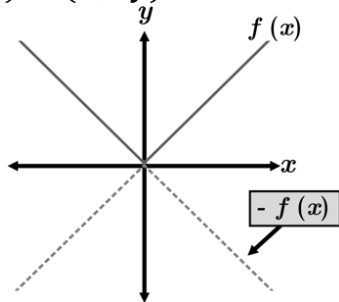
Point: $(x, y) \rightarrow (x \pm a, y)$



Reflection in x -axis:

Function: $f(x) \rightarrow -f(x)$

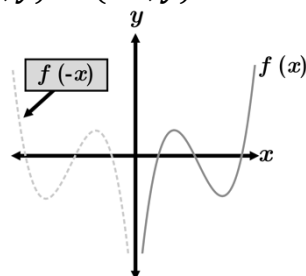
Point: $(x, y) \rightarrow (x, -y)$



Reflection in y -axis:

Function: $f(x) \rightarrow f(-x)$

Point: $(x, y) \rightarrow (-x, y)$

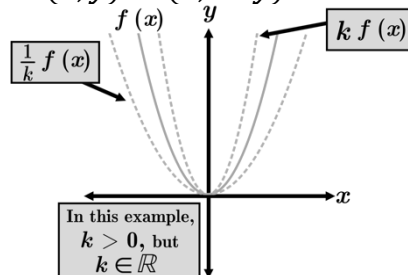


Vertical Scaling:

Function: $f(x) \rightarrow k \cdot f(x)$

and $f(x) \rightarrow \frac{1}{k} \cdot f(x)$

Point: $(x, y) \rightarrow (x, k \cdot y)$

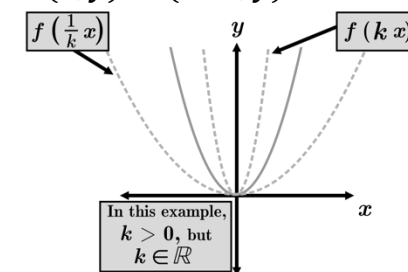


Horizontal Scaling:

Function: $f(x) \rightarrow f(k \cdot x)$

and $f(x) \rightarrow f(\frac{1}{k} \cdot x)$

Point: $(x, y) \rightarrow (k \cdot x, y)$



Identifying Transformations using HDRV

If you're asked to identify the transformations used on a function, use the following acronym in the exact order listed:

HDRV → **H**elicopters **D**o **R**ise **V**ertically

H → Horizontal translation D → Dilation R → Reflection V → Vertical translation

Example: Given the parent function $f(x) = \sqrt{x}$, describe the transformations used to result in the equation $g(x) = -2\sqrt{x-4} + 3$.

Answer: A horizontal shift to the right by 4 units, followed by a dilation with a scale factor of 2, followed by a reflection over the x -axis, followed by a vertical shift up by three units.

Even & Odd Functions

Even Functions

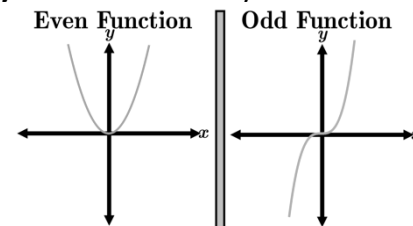
Algebraically: A function is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$.

Graphically: The function is symmetric about the y -axis.

Odd Functions

Algebraically: A function is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$.

Graphically: The function is symmetric about the origin.



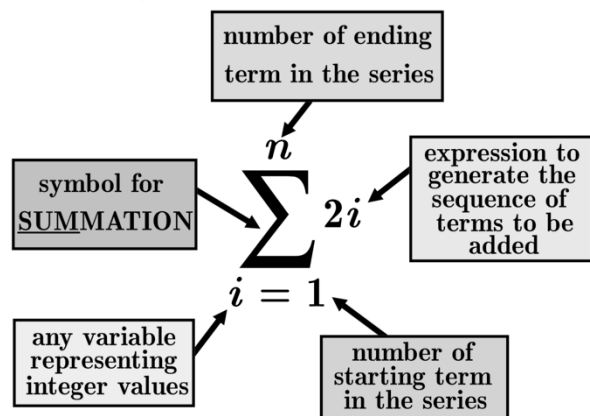
Special Cases:

- $f(x) = \sin(x)$ is an **odd** function
- $f(x) = \cos(x)$ is an **even** function
- $f(x) = \tan(x)$ is an **odd** function



SEQUENCES & SERIES

Sigma Notation: *Sigma notation* is used to write a series in a shorthand form. It is used to represent the *sum* of a number of terms having a common form. The diagram below shows the parts of a sigma notation (otherwise known as *summation*).



Example: Evaluate $\sum_{n=2}^5 (3n - 2)$
 $(3(2) - 2) + (3(3) - 2) + (3(4) - 2) + (3(5) - 2)$
 $(4) + (7) + (10) + (13) = 34$

Sums of Finite Sequences

Arithmetic Series Formula:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

where n is the number of terms in the sum, a_1 is the first term, and a_n is the n th term in the sum.

Geometric Series Formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where r is the common ratio and $r \neq 1$, n is the number of terms in the sum, and a_1 is the first term.

Important Terms & Definitions

Sequence: a list of terms or elements in order. The terms are identified using positive integers as subscripts of a : $a_1, a_2, a_3, \dots, a_n$. The terms in a sequence can form a pattern or they can be random.

Series: the sum of all the terms of a sequence.

Explicit Formula: If specific terms are not given, a formula, sometimes called an explicit formula, is given. It can be used by substituting the number of the term desired into the formula for " n ".

Recursive Formula: In a recursive formula, the first term in a sequence is given and subsequent terms are defined by the term before it. If a_n is the term we are looking for, a_{n-1} , which is the term *before* a_n , must be used.

Formulas

Remember!

Common Difference (d): $a_2 - a_1$

Common Ratio (r): $\frac{a_2}{a_1}$

	Arithmetic Sequences	Geometric Sequences
Explicit Formula	$a_n = a_1 + (n - 1)d$ <p>where "a_1" is the first term of the sequence, "n" is the desired term, and "d" is the common difference.</p>	$a_n = a_1 \cdot (r)^{n - 1}$ <p>where "a_1" is the first term of the sequence, "n" is the desired term, and "r" is the common ratio.</p>
Recursive Formula	$a_1 = ?$ $a_n = a_{n - 1} + d$ <p>where "a_1" is the first term of the sequence, "n" is the desired term, and "d" is the common difference.</p>	$a_1 = ?$ $a_n = a_{n - 1} \cdot r$ <p>where "a_1" is the first term of the sequence, "n" is the desired term, and "r" is the common ratio.</p>



STATISTICS & INFERENCE

Types of Statistical Studies

Survey: used to gather large quantities of facts or opinions. Surveys are usually asked in the form of a question, like questions from the TV show *Family Feud*.

For example, “Do you like Algebra, Geometry, or neither?” would be a survey question.



Observational Study: the observer does not have any interaction with the subjects and just examines the results of an activity. For example, the location as to where the Sun rises and sets on each day throughout the year would be an observational study.

Controlled Experiment: two groups are studied while an experiment is performed with one of them but not the other. For example, testing if orange juice has an effect in preventing the “common cold” with a group of 100 people, where 50 people will drink orange juice and the other 50 will not drink the juice. The statistician will then analyze the data of the control group and the experimental group. The conclusions will be the result of a controlled experiment.

Important Definitions & Terms

These are some definitions that appear most frequently on the Algebra 2 Regents. If you need to refresh your memory on *basic* terms, such as median, mode, interquartile range, etc., then you should definitely review those in our Algebra 1 study guide ☺.

Biased: a data set that is obtained that is likely to be influenced by something – giving a “slant” to the results.

Un-Biased: a data set that is obtained which does *not* favor any one group over another.

Mean: the average of the data values. It is the line of symmetry of the normal curve. The symbol for the *sample mean* is \bar{x} , whereas the symbol for the *population mean* is μ .

Variance: the average of the squared differences the data points are from the mean. The symbol is s^2 .

Standard Deviation: a measure of the spread of the data. It is the square root of the variance. Standard deviation of a *sample* is represented by the symbol s , and of a *population*, represented by the symbol σ .

Statistical Inference: draws conclusions about a population, based on data from a sample.

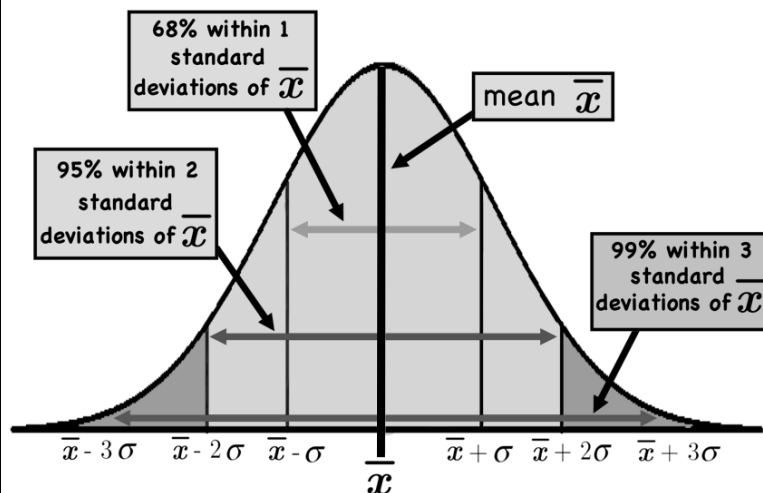
Parameter: a characteristic or measure obtained by using data from a *population*

Point Estimate: a point estimate of a parameter is the value of a statistic used to estimate the parameter. It is typically denoted as \hat{p} , called “*p hat*”.

Margin of Error: the amount that a value might be above or below an observed statistic.

Confidence Level: the likelihood that the interval estimate will contain the true population parameter.

The Normal Distribution Curve



Characteristics of the Normal Distribution Curve

- Mean = Median = Mode
- A vertical line at the mean is the line of symmetry of the curve
- Approximately 68% of the data is within 1 standard deviation, i.e. $\bar{x} \pm \sigma$, of the mean. This means it can be above or below the mean
- Approximately 95% of the data is within 2 standard deviations, i.e. $\bar{x} \pm 2\sigma$, of the mean. This means it can be above or below the mean
- Approximately 99% of the data is within 3 standard deviations, i.e. $\bar{x} \pm 3\sigma$, of the mean. This means it can be above or below the mean




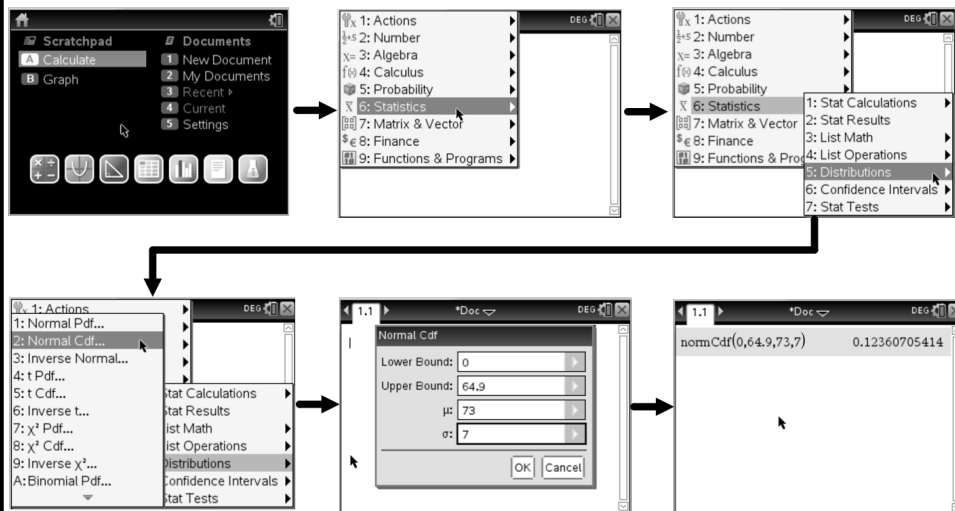
TI-Nspire CX – NormalCDF

The percentages indicated on the normal curve diagram indicate the percent of the area under the curve that is located between the indicated standard deviations and it is equal to the probability of the event occurring within those boundaries. However, when working with boundaries that are *not* on the standard deviation diagrams, a graphing calculator must be used using “NormalCDF”

Example: Using the TI-Nspire CX, find the normalCDF to the nearest hundredth if $\mu = 73$, the standard deviation ($\sigma = 7$), the lower bound is 0, and the upper bound is 64.9.

Steps on the Calculator

- 1) Open a “New Document – Calculator”
- 2) Click the “Menu” button 
- 3) Click on 6: Statistics, followed by 5: Distributions, followed by 2: Normal CDF.
- 4) Enter all required data, then click OK.



The answer is .12

Confidence Intervals

A *confidence interval* is a range or interval of values used to estimate the true value of a population parameter. The formula to calculate the confidence interval is given by:

$$C.I. = \bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Level	Z
90 %	1.645
95 %	1.96
99 %	2.575

Yes... you *must* memorize this chart, especially for the margin of error formula.

... where σ is a known value, \bar{x} is the mean, z changes value depending on the confidence level (see table above), and n is a sample population.

Z-Scores

A z-score represents the how many standard deviations a value is over or below the mean, μ . A z-score of one means the value is one standard deviation above the mean.

Formulas:

$$\text{Sample z-score: } z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x - \bar{x}}{s}$$

where x is the value being examined, \bar{x} is the sample mean, and s is the sample standard deviation.

$$\text{Population z-score: } z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

where x is the value being examined, μ is the population mean, and σ is the population standard deviation.

Notes:

- A negative z-score represents a value less than the mean
- A z-score of zero represents the mean
- A positive z-score represents a value greater than the mean

Margin of Error Formula

Definition: refers to the distance from the estimate to one end of the confidence interval.

$$M.O.E. = z \cdot \sqrt{\frac{p(1-p)}{n}}$$

Confidence Level	Z
90 %	1.645
95 %	1.96
99 %	2.575

... where z is the z-score depending on the given confidence level (see chart above), p is the mean of the sampling proportion, and n is the sample size.



SET THEORY & PROBABILITY

Venn Diagrams & Set Relationships

Set Theory – Brief Overview

Set theory is an important tool in the study of probability, as well as future mathematics. You will need to understand the definitions, terminology, and symbols associated with set theory. This information was most likely not covered in your class, but should have been taught for better understandings of symbols & concepts.

Definitions

Universal Set: the set of all possible elements available to form subsets. The universal set is normally denoted as u .

Set: a group of specific terms within the universal set u .

Subset: a set whose elements are completely contained within a larger or equally sized set.

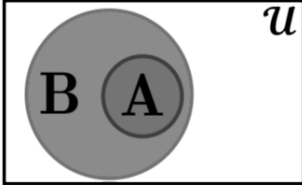
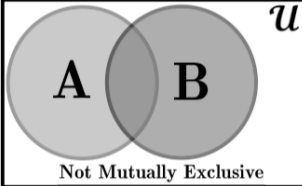
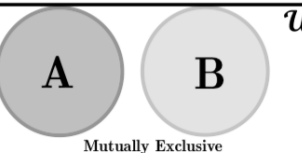
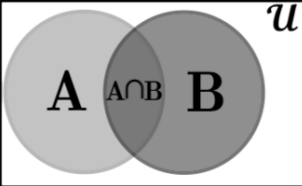
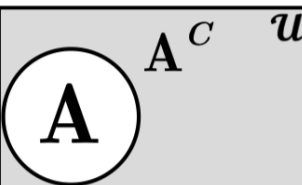
Complement of a Set: contains the elements of the universal set that are not in the originally defined set.

Mutually Exclusive: two sets that have no elements in common are called mutually exclusive sets.

Null Set: otherwise known as the empty set; contains no elements

Common Symbols

Symbol	English Meaning
\in	"is an element of"
\cup	Union
\cap	Intersection
\subseteq	Subset
A' or \bar{A} A^c or $\sim A$	Complement
\emptyset { }	Null Set

Set Notation	English Meaning	Venn Diagram
$A \subseteq B$	Subset: A is a subset of B . All the elements in A are also in B	
$A \cup B$	Union of two sets is the set of elements in either A or B , or in both	
$A \cup B$	Union of two sets is the set of elements in either A or B	
$A \cap B$	Intersection of two sets is the set of elements that are in both sets A & B	
A' or \bar{A} or A^c or $\sim A$	Complement of a set is the elements that are in the universal set, but not in the given set	



Definitions & Types of Probabilities

Probability Values:

Suppose we have an event E . Then the following hold to be true:

- $P(E)$ is never less than zero or more than one. That is, $0 \leq P(E) \leq 1$.
- $P(E) = 0$ when the event is not possible to occur.
- $P(E) = 1$ when the event is certainly possible to occur.

Complement of $P(E)$: the probability that E does *not* happen is $1 - P(E)$, denoted as $P(E')$.

Mutually Exclusive Events: two events that have no outcomes in common.

Independent Events: two events are independent if the outcome of one event does not change the probability of the other event.

Dependent Events: the outcome of one event impacts the probability of the other event.

Conditional Probability: $P(A | B)$ is read as “the probability of A given B ”. It means the probability of event A occurring after event B occurred.

Types of Probabilities:

Probabilities can be calculated in different ways:

- **Theoretical Probabilities:** probabilities come from assumptions about an event and its outcomes.
- **Empirical Probabilities:** probabilities come from data on many observations or trials.
- **Simulation:** probabilities that are based on data from a model. A simulation is a model in which repeated experiments are conducted to imitate a real-world situation and produce similar results.

General Probability Rules & Definitions

**You can skip this section if you’d like, since you do not need to “know this stuff cold”. For the formulas, see the next section. **

You have already seen set theory notations. For probability, however, these mathematical symbols have different meanings.

- ✓ When you see the intersection symbol denoted as \cap , think of the word “**and**”
- ✓ When you see the union symbol denoted as \cup , think of the word “**or**”

And (Intersection, notation as \cap): the probability of two or more independent events occurring in a row, one and then the other, can be found by multiplying the individual probabilities. $P(A \cap B) = P(A) \cdot P(B)$.

Or (Union, notation as \cup): two situations can occur when dealing with the union of two probabilities. The events can be *mutually exclusive* (no common or overlapping outcomes) or *not mutually exclusive* (some common outcomes). There is a general formula for the union that is adjusted and applied to this type of problem. This formula is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If, however the events *are mutually exclusive*, then $P(A \cap B) = 0$ and the formula changes to $P(A \cup B) = P(A) + P(B)$.

Conditional (Given) Probability $P(A | B)$: conditional probability notation is used when one event happens after a given event has already occurred. $P(B | A)$ is read as, “the probability of event A occurring after event B has already occurred”. The formula for the conditional probability is $P(B | A) = \frac{P(A \cap B)}{P(A)}$ or $\frac{P(A | B)}{P(B)}$.

Independent Probability of Events: the probability of two independent events occurring in sequence is the product of their individual probabilities. This is called the Multiplication Rule, and multiplication is represented as “and”. If two events are independent, then the probability of them both occurring is $P(A \cap B) = P(A) \cdot P(B)$.

Dependent Probability of Events: for dependent events, we can arrange the conditional probability formula to get $P(A \cap B) = P(A) \cdot P(B | A)$.

Probability Formulas

Here are *all* of the probability formulas that you *must know cold* for the regents. These will not be given to you, so **know them!** Let A and B be events. Then the following hold true:

Conditional (OR) Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proving Two Events are Independent:

Formula 1: $P(A \cap B) = P(A) \cdot P(B)$

Formula 2: $P(A) = P(A | B)$

Formula 3: $P(B) = P(B | A)$

You only need to use one of these formulas, but chances are, you are going to be using the first one on your regents!



Proving Two Events are Dependent: $P(A \cap B) = P(A) \cdot P(B | A)$

Events A and B are Mutually Exclusive: $P(A \cap B) = 0$



APPLICATIONS OF LOGARITHMS & REGRESSION

Regression Models with the TI – Nspire CX Calculator

You have seen regression models in Algebra 1. Review the

Exponential Growth & Decay

When a given quantity is increased or decreased overtime by a certain percentage, we can calculate the anticipated results using one of the two formulas below:

Exponential Growth

$$A(t) = P(1 + r)^t$$

... where $A(t)$ represents the total amount, P is the initial/principal/starting amount, r is the rate on increase for a specific time expressed as a decimal, and t is time.

Exponential Decay

$$A(t) = P(1 - r)^t$$

... where $A(t)$ represents the total amount, P is the initial/principal/starting amount, r is the rate on decrease for a specific time expressed as a decimal, and t is time.

Compound Interest with Logarithmic Applications

Compound Interest occurs when the principal invested at a given rate per year is compounded a specific number, n , of times per year and each time the interest is calculated, the amount of the interest is added to the present value (originally the principal).

Compound Interest Formula

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

... where A is the total amount of dollars, P is the principal, r is the annual rate expressed as a decimal, n is the number of compounds per year, and t is the time.

Understand that n changes according to “buzz words”, shown in the chart to the right.

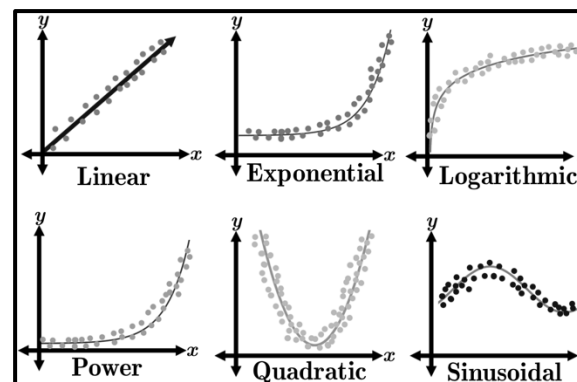
Compound Continuously Formula

$$A = Pe^{rt}$$

... where A is the total amount of dollars, P is the principal, r is the annual rate expressed as a decimal, and t is the time.


English Phrase	What n Equals
Compounded Annually	$n = 1$
Compounded Quarterly	$n = 4$
Compounded Monthly	$n = 12$
Compounded Daily	$n = 365$

basics if you have to by glancing at our Algebra 1 study guide. Most likely, you are most familiar with generating a regression model for equations of lines. A regression equation is a function that represents the graph of a line or curve of best fit. You can use your TI – Nspire CX to develop necessary values to input into a general equation, as well as view a picture of the regression. Here are some examples:



The forms of these equations can be accessed on the TI – Nspire CX graphing calculator. Steps are listed below to access these regression templates, allowing you to run accurate representations of a given data set.

Steps on the Calculator

- 1) Open a “New Document – Calculator”
- 2) Click the “Menu” button 
- 3) Click on 6: Statistics, followed by 1: Stat Calculations, followed by option 3, 6, 9, A, B, or C.
- 4) Enter all required data, then click OK.

